1. PHYSICAL and MATHEMATICAL MODELLING of RADIAL POLYTRON

1.1. Methods and outcomes of physical modelling.

The physical modelling of polytrons (both radial, and axial) was fulfilled by excitation of mechanical resonant oscillations in rings of metal wire and fillet.

The excitation of oscillations in rings was fulfilled by electromagnetic method by means of generator of signals of low frequency Г3-109. For manufacturing of rings the widespread materials were taken. A wire and fillet of carbon steels, which are used in different spring gears, and also, brass, copper and aluminium wire from the bill of rating of electrical industry. At excitation of oscillations in steel rings, the variable harmonic current from the generator was flowed through an electromagnet, in an air gap of which one small segment of the ring was placed. At excitation of oscillations in rings from non-magnetic materials, the variable harmonic current was flowed in the ring, and the small segment of the ring was placed in an air gap of permanent magnet.

Besides, a series of experiences on excitation of resonant oscillations in the same rings by an only mechanical way was fulfilled. In these experiences an alternating current from the generator was flowed in the electromechanical vibrator, anchor of which one, in the form of mild hollow rod, was placed inside of a cylindrical coil. One end of anchor was placed inside the coil, and to its back the tablet of a high-coercive magnet was pasted. The second end of anchor acted from the coil and was supplied with a miniature clamp for attachment of investigated rings.

At excitation of oscillations with the help of the electromechanical vibrator the polytrons with only odd frequency order ($\boldsymbol{m}$ = 3, 5, …) are modelling, since one node of polytron is connected with clamp.

This series of experiences was fulfilled with the purpose of detection of effect of a way of excitation of oscillations on measured values of resonance frequencies of investigated rings. In outcome, the same values of resonance frequencies of odd polytrons were obtained precisely, as well as at excitation of oscillations by electromagnetic method.

At realization of experiences with rings of a steel wire and a steel fillets its were compared resonance frequencies of rings of identical diameter, which were made of a steel wire of diameter 0.2 mm and of a steel fillet of thickness 0.2 mm and width 5 mm. Besides, the two-layer and three-layer rings minimized in a roll and welded by a spot welding in one place were made of a steel fillet. The resonance frequencies of single-layer rings of a steel fillet have appeared little bit below, than resonance frequencies that of the frequency order m of wire rings. It is possible to explain this minor decreasing of frequencies by a more essential air resistance for a flat surface of a fillet and, arising thus, effect of an added mass.

As to resonance frequencies of one-, two- and three-layer rings of a steel fillet, they practically do not differ. Each layer in a ring behaves as an independent ring, and as the diameters of these layers-rings are almost peer, also resonance frequencies of them are indiscernible. However, width of resonant spikes in rings with miscellaneous number of layers is proportional to number of layers and this fact allows to compare mechanical oscillations in the ring-shaped forms to electrical oscillations in the radio schemes and to apply units of the theory of electrical oscillations.

For approximated physical modelling of free polytrons, i.e. of polytrons with even frequency order only, the rings of a steel wire were set on four or three racks of a thin wire so that the nodes of resonance ring were placed on racks. The top ends of a wire racks were bent by a way character “φ”, that provided for ring a small radial backlash in nodes. In such a way, it was possible to reduce to minimum the influence of units of attachment on resonance frequencies of investigated rings.

As a result of these experiences it was established, that the qualitative character of dependence of resonance frequencies of rings from the frequency order m remains invariable, but in a quantitative relation, resonance frequencies of free rings are lower, than identical frequencies of rings, which are fastened in one node. To explain this fact, it is possible, the rings, which are fastened in one node, have near this node an increased rigidity, which gives rise to increasing of resonance frequencies. This explanation still by such experimental fact is confirmed, that quantrons, directly outgoing from the fastened node, have smaller angular distance between nodes, than following behind them quantrons.

In Fig.2 three pairs of experimental curves, which reflect the dependence of resonance frequencies from the frequency order for fastened rings (continuous lines) and free rings (dotted lines) are shown.

As it is visible from Fig.2 the resonance frequency of ring and the frequency order m are connected by quite particular parabolic relation, and essentially depend on relation of diameter of a wire ***d*** to diameter of ring ***D***. At rather small relation ***d/D***, the frequencies become practically equal and, therefore, in calculations one frequency will be used only.

The amplitude of radial oscillations of ring, to be exact to tell, amplitude of radial quantrons, also depend on the frequency order ***m***, and the quantum character of this relation is quite obvious - the less angular distance between nodes, the should be and amplitude, at the same power of swing, made into ring. This relation has hyperbolic character and especially clearly appears at achievement of limit value of the radial amplitude, which correspond to the given frequency order ***m***.

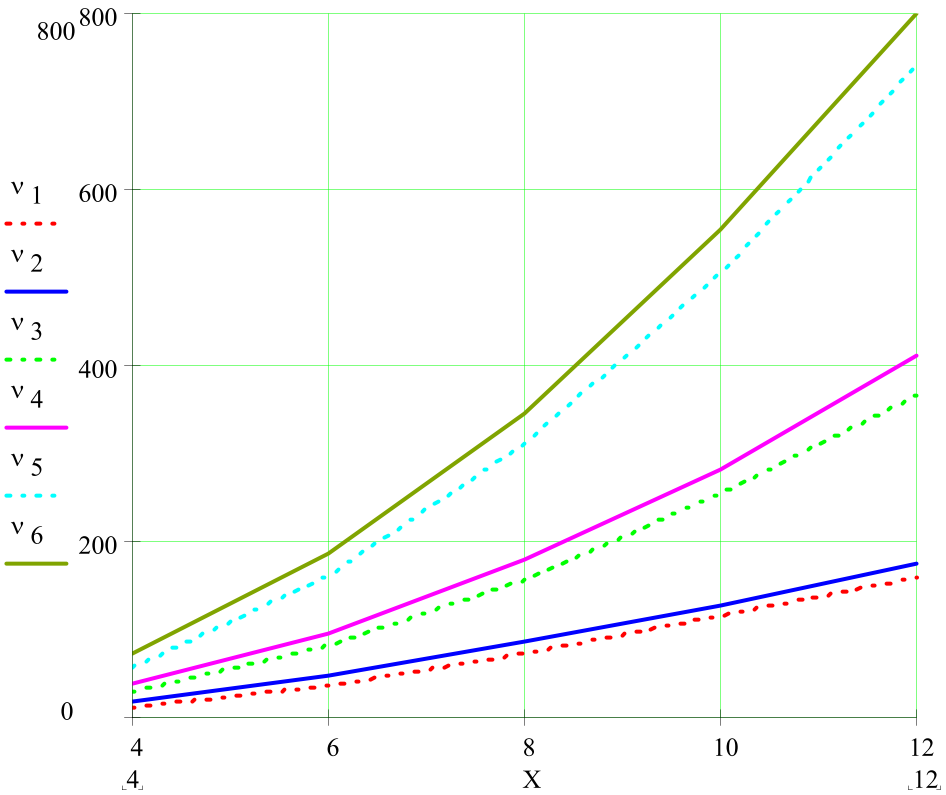

Fig. 2

Graphic chart of resonance frequencies from the frequency order of free and fixed steel rings at ratio d/D equal: $4{,}56 \cdot 10^{-3}$- upper pair of curves, $2{,}28 \cdot 10^{-3}$ - middle pair of curves, $1{,}14 \cdot 10^{-3}$- lower pair of curves.

As to relation of radial amplitude to a power of swing, experimentally was established, that the curve of this relation is similar to an abrupt exponent with fast accessible limit.

During experiments, the impact supply power of swing and step change its was applied also. At such ways of impact supply power of swing, the radial amplitude smoothly leaves the limit value, which corresponds to stationary mode of supply power of swing, but fast returns to value of this limit.

And, at last, after the radial amplitude will reach the limit value, the further repeated increase of power input to the ring, does not influence value of limit amplitude at all, and the redundant power will be converted into heat and into energy of electromagnetic radiation with frequency, equal resonance frequency of oscillations of ring.

The qualitative analysis of all fulfilled researches allows making following conclusions:

1) A series of resonance frequencies of ring oscillator characterized by the frequency order ***m***, essentially differs from similar relation for a linear harmonic

oscillator. Increase of resonance frequencies and increase of the frequency order ***m*** are connected by parabolic relation.

2) The resonance frequencies of radial oscillations of a ring are linearly proportional to a longitudinal speed of elastic shift deformations in a ring, are linearly proportional to thickness of a ring in a radial direction, and are inversely proportional to a square of diameter of a ring.

3) The amplitude of radial oscillations of ring has the upper energy limit, which one is connected with the frequency order ***m*** by hyperbolic relation and has discrete character.

4) In the field of energies, which is lower than the energy limit, the radial amplitude of quantrons depends on a total energy of ring and is change exponentially. Probably, as in this area, the radial amplitude is change discretely, but necessarily is integer.

1.2 Mathematical modelling of radial quantoide.

At radial oscillations polytronic quantoide is arranged in one plane, therefore its equation the most conveniently to record in polar coordinates. The position of any point of radial quantoide is determined by six arguments - static diameter of polytron $\boldsymbol{D_s}$, its frequency order ***m***, radial amplitude order $\boldsymbol{n_r}$, polar angle ***φ*** of polar coordinates, frequency of resonant oscillations ***ν*** and time ***t***

$$\rho(m,n_r,t,\phi)=\frac{D_s\cdot\sqrt{m^2+n_r^2\cdot\cos\left(\frac{m\cdot\phi}{2}\right)\cdot\left|\cos\left(\frac{m\cdot\phi}{2}\right)\right|\cdot\cos(2\cdot\pi\cdot\nu\cdot t)}}{2\cdot m\cdot\left[\frac{64\cdot\left(m^2+4\right)+n_r^2\cdot(\cos(2\cdot\pi\cdot\nu\cdot t))^2}{64\cdot\left(m^2+4\right)}\right]} \tag{1-1}$$

where: $\boldsymbol{\rho(m, n_r, t, \varphi)}$ - polar radius-vector from center of coordinates to point of quantoide, appropriated to a polar angle φ in the given instant t. At ***t*** = 0 radius-vector (1-1) describe the boundary quantoide.

Length of the boundary quantoide is calculated by integration from 0 up to 2π under the formula

$$l_r(m,n_r,\phi)=\int_0^{\phi}\sqrt{\rho(m,n_r,\phi)^2+\rho'(m,n_r,\phi)^2}\,d\phi \tag{1-2}$$

where: $\boldsymbol{\rho'(m,n_r,\varphi)}$ - first derivative of a radius-vector $\boldsymbol{\rho(m, n_r, t, \varphi)}$ on the angle ***φ***.

$$\rho'(m,n_r,\phi)=\frac{D_s\cdot n^2\cdot(-\sin(m\cdot\phi))}{8\cdot\left[\frac{64\cdot(m^2+4)+n_r^2}{64\cdot(m^2+4)}\right]\cdot\sqrt{m^2+n_r^2\cdot\cos\left(\frac{m\cdot\phi}{2}\right)\cdot\left|\cos\left(\frac{m\cdot\phi}{2}\right)\right|}}$$

(1-3)

Each point of quantoide is characterized by one more angular parameter connected with polar angle φ. It is an angle between tangent to quantoide in the given point and normal to a polar radius-vector in this point.

$$\psi(m,n_r,\phi)=\operatorname{arctg}\frac{\rho'(m,n_r,\phi)}{\rho(m,n_r,\phi)} \qquad (1\text{-}4)$$

Using the formula (1-4) and by expressing a tangent of an angle through its cosine, we shall receive other kind of the formula (1-2)

$$l_r(m,n_r,\phi)=\int_0^{\phi}\frac{\rho(m,n_r,\phi)}{\cos(\psi(m,n_r,\phi))}d\phi \qquad (1\text{-}5)$$

In frame of the formula (1-1) there are dynamic diameter of a radial polytron $\boldsymbol{D_r}$ and both amplitudes of points of quantoide – radial and tangential. And because of an asymmetry of amplitudes of external and internal points of quantoide, they need to be studied separately.

$$D_r=D_s\cdot\frac{64\cdot(m^2+4)}{64\cdot(m^2+4)+n_r^2} \qquad (1\text{-}6)$$

$$a_p(m,n_r,\phi)=\frac{D_r}{2}\cdot\left(\sqrt{1+\left[\frac{n_r}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)\right]^2}-1\right) \qquad (1\text{-}7)$$

$$a_q(m,n_r,\phi)=\frac{D_r}{2}\cdot\left(1-\sqrt{1-\left[\frac{n_r}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)\right]^2}\right) \qquad (1\text{-}8)$$

$$\tau_p(m,n_r,\phi)=\frac{D_r}{2}\cdot\operatorname{tg}\left(\frac{2\cdot l_p(m,n_r,\phi)}{D_s}-\phi\right) \qquad (1\text{-}9)$$

$$\tau_q(m,n_r,\phi)=\frac{D_r}{2}\cdot\operatorname{tg}\left(\phi-\frac{2\cdot l_q(m,n_r,\phi)}{D_s}\right) \qquad (1\text{-}10)$$

where:

$a_p(m,n_r,\varphi)$ – radial amplitude of external points of quantoide

$a_q(m,n_r,\varphi)$ – radial amplitude of internal points of quantoide

$\tau_p(m,n_r,\varphi)$ – tangential amplitude of external points of quantoide

$\tau_q(m,n_r,\varphi)$ – tangential amplitude of internal points of quantoide

$$l_p(m,n_r,\phi) = \int_0^\phi \sqrt{\rho_p(m,n_r,\phi)^2 + \rho'_p(m,n_r,\phi)^2}\, d\phi$$

$$l_q(m,n_r,\phi) = \int_0^\phi \sqrt{\rho_q(m,n_r,\phi)^2 + \rho'_q(m,n_r,\phi)^2}\, d\phi$$

$$\rho'_p(m,n_r,\varphi) = \frac{D_r \cdot n_r^2 \cdot (-\sin(m\cdot\varphi))}{8\cdot\sqrt{m^2 + \left[n_r \cdot \cos\left(\frac{m\cdot\phi}{2}\right)\right]^2}}$$

$$\rho'_q(m,n_r,\varphi) = \frac{D_r \cdot n_r^2 \cdot (\sin(m\cdot\varphi))}{8\cdot\sqrt{m^2 - \left[n_r \cdot \cos\left(\frac{m\cdot\phi}{2}\right)\right]^2}}$$

In Fig.3 the geometrical constructions are fulfilled, which illustrate a principle of calculation of radial and tangential amplitudes and indicate to scale a ratio of the sizes in the polytron PT√2/3R.

The angular displacements of points from its positions on static diameter are peer:

$$\varphi_p = 2\cdot l_p(m,n_r,\varphi)/D_s - \varphi \quad \text{and} \quad \varphi_q = \varphi - 2\cdot l_q(m,n_r,\varphi)/D_s$$

In Fig.4 the relations of radial and tangential amplitudes of a polytron PT√2/3R to a polar angle φ are adduced. The continuous lines (greasy and thin) concern to radial amplitudes, dotted lines (greasy and thin) - to tangential amplitudes.

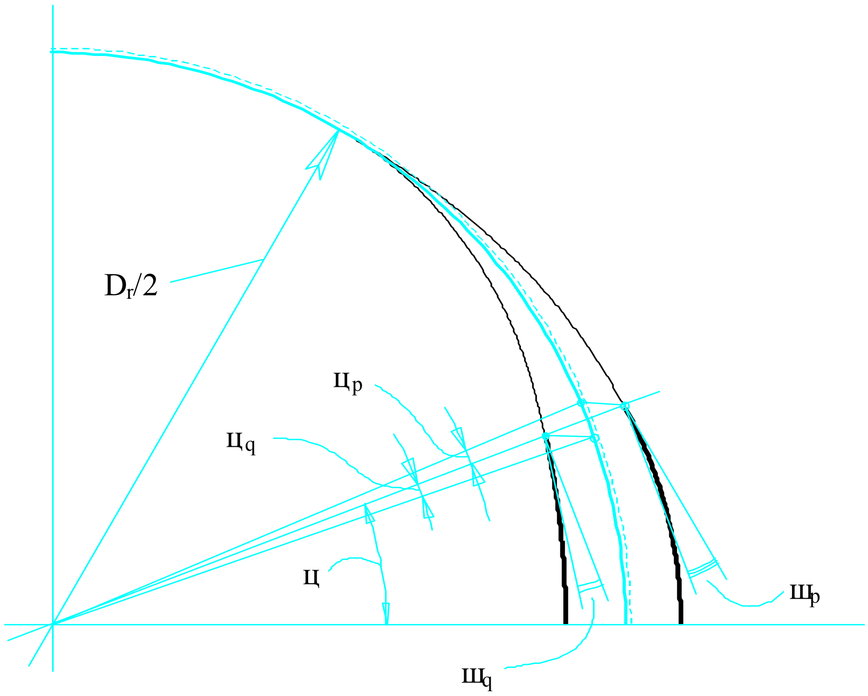

Fig. 3
Graphic explanations to a principle of calculation of radial and tangential amplitudes in polytron. The dotted line shows the static diameter of polytron.

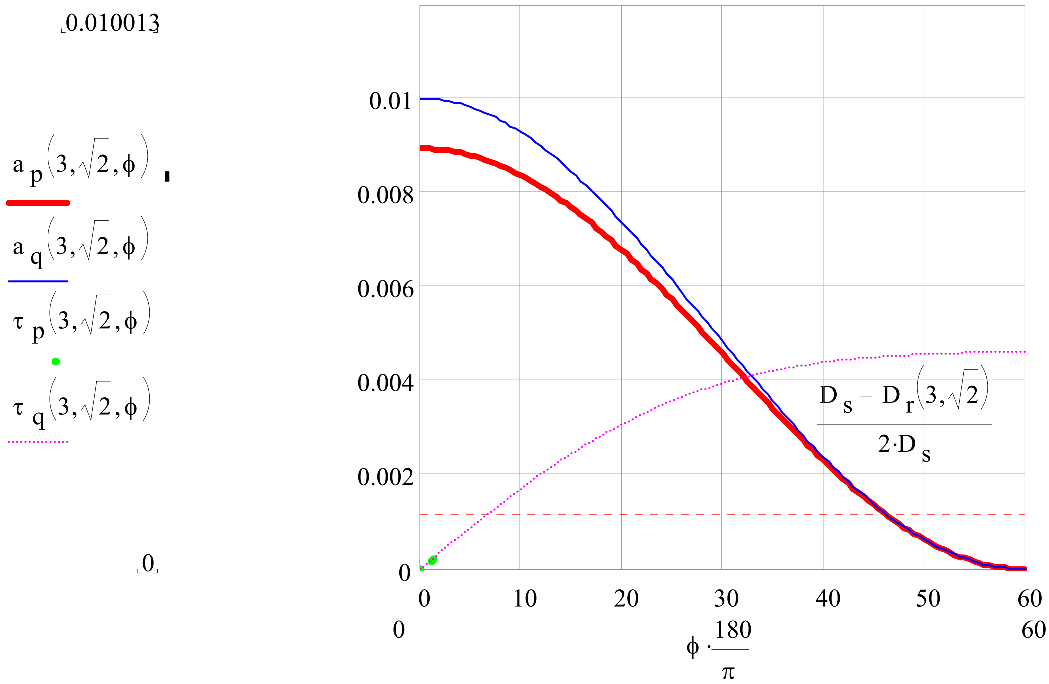

Fig. 4
Graphic chart of radial amplitudes (continuous lines) and tangential amplitudes (dotted lines) from a polar angle φ for boundary quantoide of polytron
PT√2/3R.
The horizontal dotted line marks a position of static diameter of polytron.

At radial oscillations, the ends of an internal part of quantoide of one quantron pass through nodes into sectors, which belong to an external part of quantoide of adjacent quantron. Besides, the points commit also transitions from static diameter into dynamic one and back. The frequency of these transitions is twice higher than frequency of oscillations of quantoide. In outcome, there is a rather complex process of exchange of energy between external and internal halves of quantrons.

Space between static and dynamic diameters of a polytron is named as a dynamic layer and is characterized by value, calculated under the formula

$$d_r = \frac{D_s}{2} \cdot \left(\frac{n_r^2}{64 \cdot (m^2 + 4) + n_r^2} \right) \qquad \text{(1-11)}$$

Thus, parameter dr is a double amplitude of the dynamic layer, which pulsating with doubled frequency.

The total energy of oscillations of each point of quantoide is proportional to the sum of squares of its radial and tangential amplitudes. Energy of a point connected to its oscillations in a dynamic layer is so small, that practically does not influence on a total energy of a point.

$$E_p(m, n_r, \phi) \sim a_p^2(m, n_r, \phi) + \tau_p^2(m, n_r, \phi)$$

$$E_q(m, n_r, \phi) \sim a_q^2(m, n_r, \phi) + \tau_q^2(m, n_r, \phi)$$

$$E_d(m, n_r) \sim d_r^2$$

In Fig.5 the curves, which evaluate relative values of a total energy of each external point of quantoide $E_p(m,n_r,\varphi)$ and of each internal point of quantoide $E_q(m,n_r,\varphi)$ depending on an angular position of this point in polytron $PT\sqrt{2/3}R$ are shown. The horizontal dotted line marks relative value of energy of oscillations of this point in dynamic layer multiplied by 100.

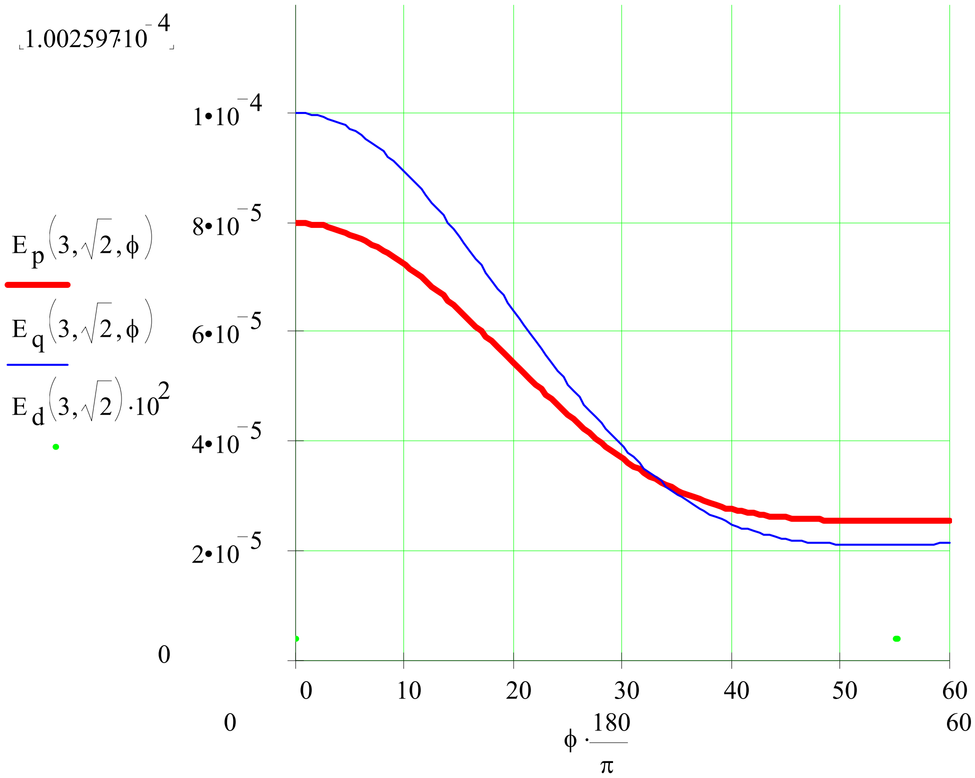

Fig. 5

Graphic chart of energy of oscillations of points of quantoide from a polar angle φ:
heavy line - a total energy of external points of quantoide,
light line - a total energy of internal points of quantoide.

From Fig.5 it is visible, that near of nodes, the points of quantoide have minimum and practically constant energy. It testifies that these points of polytron move practically linearly.

The formula (1-1) responds the condition of persistence of length of quantoide to accuracy within the 100-th shares of percent and, besides, has two all-important properties, namely:

1) The area each radial quantron is divided by the line of the circle of dynamic diameter into two absolutely equal parts;

2) If the amplitude order of polytron is preset by the harmonic series

$$n_r = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots$$

then the area of the circle of dynamic diameter contains an integer of the areas of quantrons, appropriate to the given frequency order.

The area of radial quantron is calculated under formula

$$q_r = \frac{\pi \cdot D_r^2 \cdot n_r^2}{4 \cdot m^3}$$

The sum of areas of all quantrons is named as the radial dynamic area of polytron and in ***m*** times more than area of one radial quantron:

$$Q_r = m \cdot q_r = \frac{\pi \cdot D_r^2}{4} \cdot \frac{n_r^2}{m^2} \qquad (1\text{-}12)$$

In Fig.6 the continuous lines show two quantoides of polytron **PTn$_r$/4R** with the amplitude order **n$_r$** $= \sqrt{2}$ in instants biased on half-cycle of frequency of natural oscillation. Dotted lines show the same quantoides, but with the amplitude order **n$_r$** = **2**.

The experience displays, that the oscillations with amplitudes, which are shown by dotted lines, are impracticable. Sooner there will be frustration of resonant oscillations, than the amplitude will reach value close to $n_r \approx 1{,}5$.

The given experimental fact indicates that in a mode of stable oscillations, the curvature of quantoide cannot receive negative values. Nevertheless, the condition of persistence of length of quantoide is executed and at such critical amplitudes.

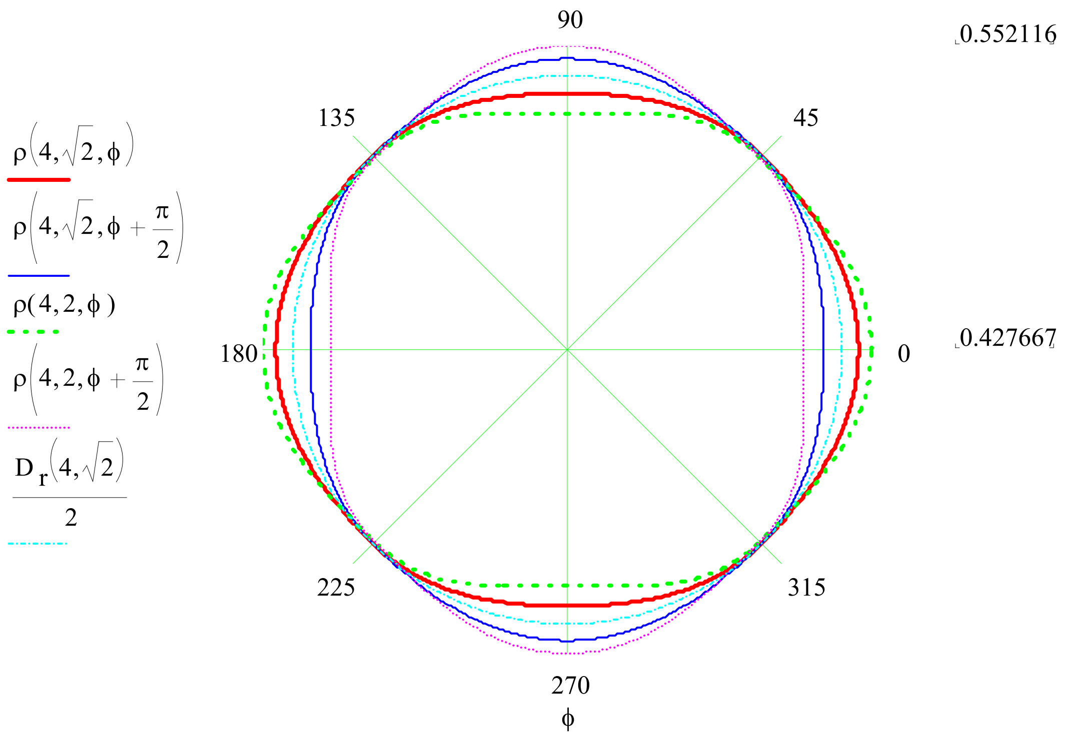

Fig. 6
The configurations of boundary quantoides of polytron PTn$_r$/4R with the amplitude orders n$_r$ = $\sqrt{2}$ and n$_r$ = 2, out-of-phase on halfcycle of resonant oscillations.

1.3 Mathematical modelling of resonance frequencies.

The creation of the formula for resonance frequencies of polytron is complicated by that circumstance, that, according to the energetic postulate, quantoide has not thickness, and, therefore, should not have such mechanical characteristic, as rigidity. In real experiments, the effect of influence of thickness of ring on values of resonance frequencies is most essential, since the thickness of ring are connected with its mass and rigidity. In order decrease a rigidity of rings it is possible, basically, at the expense of decreasing relation of diameter of wire to diameter of ring. The value, which was reached in experiments, equal ~10-3. At values of this relation (2-3)·10-3 and lower the resonance frequencies of radial and axial vibrations of rings coincide in a rather broad interval of frequencies. This circumstance has helped to foresee frame of the formula for resonance frequencies of polytron and to calculate a series of factors, included in it.

The empirical formula for calculation of resonance frequencies of rings from materials of round cross-section has following frame

$$\nu_v = \frac{v \cdot d}{8 \cdot \pi \cdot D^2} \cdot (m - k_m) \cdot \left(m + \sqrt{m^2 - k_n^2} \right) \qquad \text{(1-13)}$$

where **v** – longitudinal speed of spread of elastic shift deformations in a ring

d – diameter of a material of a ring

D – diameter of a ring (in this case static diameter of polytron)

$\mathbf{k_m}$ – factor, which is taking into account linear changes of the characteristics of rings (such, for example, as change of a modulus of elasticity under bending wires and respective alteration of speed v).

$\mathbf{k_n}$ – factor, which is taking into account non-linear changes of the characteristics of rings (such, for example, as change of rigidity depending on wire diameter, structure of material and air resistance).

The factors $\mathbf{k_m}$ for radial and axial vibrations have small difference dependent on rigidity of material of a ring in that and the other direction. At decreasing of rigidity, whether it is at the expense of reduction of relation ***d/D*** or at the expense of selection of a softer material, the correction $\mathbf{k_m}$ aims at null.

The frequency order of polytrons has low limit $\boldsymbol{m} = 3$ for polytrons fixed in one node, and $\boldsymbol{m} = 4$ for free polytrons. Therefore, factor $\mathbf{k_n}$ cannot exceed indicated values for these types of polytrons. Differently, frequency $\boldsymbol{\nu_v}$ will become a complex number.

The special case is the value of the frequency order $\boldsymbol{m} = 2$. In this case, on length of quantoide the only one wave has place and origin of resonance is theoretically probably. However, observed in experiments with metal rings the resonant oscillations with $\boldsymbol{m} = 2$, represent itself oscillations of the center of mass of ring relative to the fastening point of ring. Thus, diameter of ring does not change, i.e. dynamic diameter is absent. Frequency of oscillations of ring depends on the mass and rigidity of all units of construction in fastening point of ring. Therefore, measured resonance frequency of ring at $\boldsymbol{m} = 2$, can deviate from frequency, which appropriate under formula (1-13).

Nevertheless, the resonance at $\boldsymbol{m} = 2$ in any kind exists and, therefore, for maximum value of factor, it is necessary to accept $\mathbf{k_{n\,max} = 2}$.

Thus, the lower resonance frequency of polytron is arranged in frequency band, which is differing twice:

$$\nu_{min} = \frac{v \cdot d}{8 \cdot \pi \cdot D^2} \cdot m^2 = \frac{\nu_{max}}{2}$$

where: $\mathbf{k_n = 2}$ for $\boldsymbol{\nu_{min}}$ and $k_n = 0$ for $\boldsymbol{\nu_{max}}$

Taking into account this circumstance, it is possible to express a resonance frequency of polytron through some generalized factor $\mathbf{k_o}$, the value which one lays within the limits from 1 up to 2.

$$\nu(m) = \frac{v \cdot d \cdot m^2}{8 \cdot \pi \cdot D^2} \cdot k_o \qquad (1\text{-}14)$$

The factor $\mathbf{k_o}$ in this case can be some function of the frequency and amplitude orders of polytron.

1.4 Calculation of energy of oscillations of radial quantoide.

According to classic definition, the mechanical energy of oscillations of a point is peer to half of product of mass of a point on a square of cyclical frequency and on a square of amplitude of a point. Mass of a point in our case is mass of an elementary section of a ring. By expressing diameter of a wire $\boldsymbol{d}$ through diameter of a ring $\boldsymbol{D_s}$ and factor $\mathbf{k_d}$ ($\boldsymbol{d} = k_d \cdot \boldsymbol{D_s}$) and by executing indispensable transformations, we shall receive following expression for elementary mass

$$\partial M_s = \frac{\pi}{8} \cdot \beta \cdot D_s^3 \cdot k_d^2 \cdot \partial\phi \qquad (1\text{-}15)$$

where: $\boldsymbol{\beta}$ – density of material of a ring.

The cyclical frequency of oscillations of a ring is peer

$$\omega(m) = \frac{v \cdot m^2}{4 \cdot D_s} \cdot k_o \cdot k_d \qquad (1\text{-}16)$$

The integral expressions for all three energies of quantoide look like:

$$w_r(m, n_r) = \frac{3}{2} \cdot \left(\frac{M_s \cdot v^2 \cdot k_o^2}{256} \right) \cdot k_d^4 \cdot m^5 \cdot \left(\frac{64 \cdot (m^2 + 4)}{64 \cdot (m^2 + 4) + n_r^2} \right)^2 \cdot J_\rho(m, n_r)$$

(1-17)

$$w_t(m, n_r) = \frac{3}{2} \cdot \left(\frac{M_s \cdot v^2 \cdot k_o^2}{256} \right) \cdot k_d^4 \cdot m^5 \cdot \left(\frac{64 \cdot (m^2 + 4)}{64 \cdot (m^2 + 4) + n_r^2} \right)^2 \cdot J_t(m, n_r)$$

(1-18)

$$w_d(m, n_r) = 12 \cdot \left(\frac{\pi \cdot M_s \cdot v^2 \cdot k_o^2}{256} \right) \cdot k_d^4 \cdot m^4 \cdot \left(\frac{n_r^2}{64 \cdot (m^2 + 4) + n_r^2} \right)^2$$

(1-19)

Where $M_s = \frac{\beta \cdot \pi \cdot D_s^3}{6}$ – the mass of a sphere of static diameter

The integrals $\mathbf{J_\rho(m,n_r)}$ and $\mathbf{J_t(m,n_r)}$ determine the type of connection of radial and tangential energies of polytron with its geometry and sizes, in particular, with the dynamic area of radial polytron $\mathbf{Q_r}$.

$$J_\rho(m, n_r) = \frac{4 \cdot \pi}{m} - 2 \cdot \int_0^{\frac{\pi}{m}} \left[\sqrt{1 + \left(\frac{n_r}{m} \cdot \cos\left(\frac{m \cdot \phi}{2} \right) \right)^2} + \sqrt{1 - \left(\frac{n_r}{m} \cdot \cos\left(\frac{m \cdot \phi}{2} \right) \right)^2} \right] \partial\phi$$

(1-20)

$$J_t(m, n_r) = \int_0^{\frac{\pi}{m}} \left[tg^2\left(\frac{2 \cdot l_p(m, n_r, \phi)}{D_s} - \phi \right) + tg^2\left(\phi - \frac{2 \cdot l_q(m, n_r, \phi)}{D_s} \right) \right] \partial\phi$$

(1-21)

In Fig.7 the curves of functional dependences of all three energies of polytron from the frequency order ***m*** are adduced. By a heavy line is marked of a radial component of energy, light line - a tangential component and dotted line - energy of a dynamic layer, and, for demonstrating of curvature of the last, its values are given with tenfold increase. The mugs mark a curve of functional dependence of the dynamic area $\boldsymbol{Q_r}$ of polytron from the frequency order.

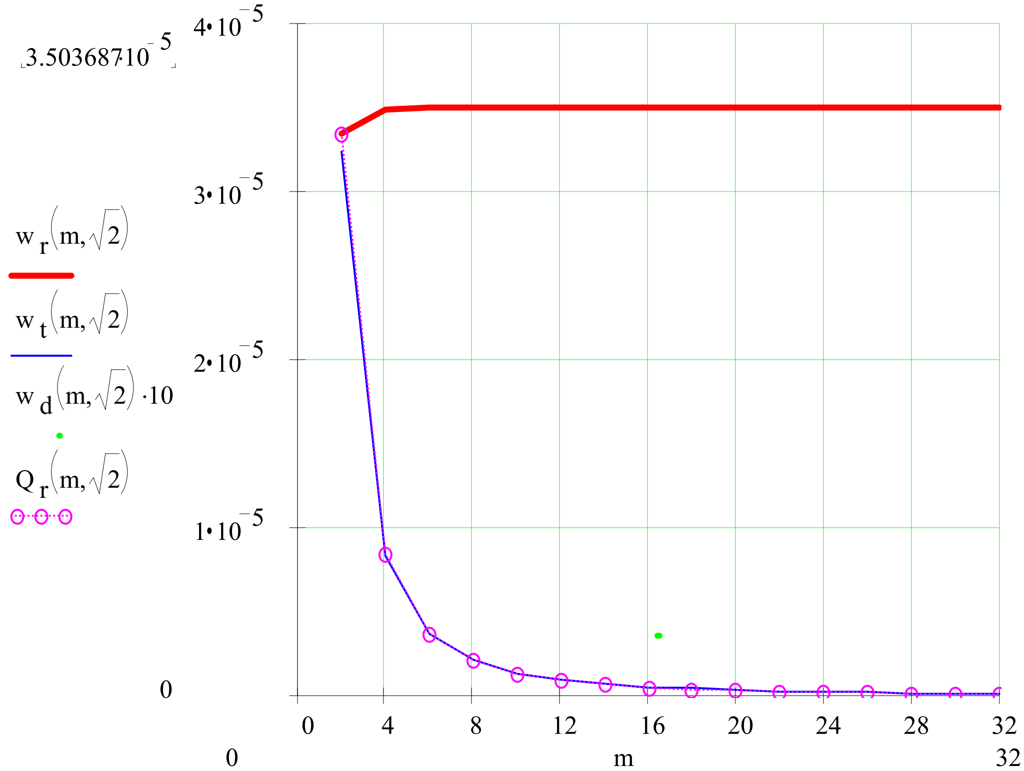

Fig. 7
Graphic chart of radial energy (heavy line), tangential energy (light line) and energy of dynamic layer (dotted line) of polytron from the frequency order m. The mugs mark relation of the dynamic area of polytron from m, reduced to a scale of a figure.

As it is visible from Fig.7, only tangential energy of polytron has precise connection with its dynamic area. The radial energy practically does not depend on the frequency order of polytron, and the energy of dynamic layer is so small, that it can be leave out at quality evaluation of the obtained outcomes.

The integrals $\mathbf{J_\rho(m,n_r)}$ and $\mathbf{J_t(m,n_r)}$ can be substituted by more simple algebraic expressions, by keeping rather high accuracy of calculations. The views of these expressions are:

$$J_\rho(m,n_r) \approx \frac{0{,}1875\cdot\pi\cdot n_r^4}{m\cdot\left(m^4+0{,}1875\cdot n_r^4\right)}\cdot\left(\frac{64\cdot\left(m^2+4\right)+n_r^2}{64\cdot\left(m^2+4\right)}\right)^2 \qquad (1\text{-}22)$$

$$J_t(m,n_r) \approx \frac{0{,}72408\cdot\pi\cdot n_r^4}{m^5\cdot\left(m^2+0{,}09\cdot n_r^2\right)}\cdot\left(\frac{64\cdot\left(m^2+4\right)+n_r^2}{64\cdot\left(m^2+4\right)}\right)^2 \qquad (1\text{-}23)$$

By substituting in expressions (1-17) and (1-18) integrals $\mathbf{J_\rho(m,n_r)}$ and $\mathbf{J_t(m,n_r)}$ by their approximated expressions, and by designating common for all three energies the part as

$$w_o = \frac{\pi \cdot M_s \cdot v^2 \cdot k_o^2}{256} \qquad (1\text{-}24)$$

let's receive the below mentioned formulas for further research of energies

$$w_r(m,n_r) = \frac{3 \cdot w_o}{2} \cdot \left(\frac{0.1875 \cdot m^4 \cdot k_d^4 \cdot n_r^4}{m^4 + 0.1875 \cdot n_r^4} \right) \qquad (1\text{-}25)$$

$$w_t(m,n_r) = \frac{3 \cdot w_o}{2} \cdot \left(\frac{0.72408 \cdot k_d^4 \cdot n_r^4}{m^2 + 0.09 \cdot n_r^2} \right) \qquad (1\text{-}26)$$

$$w_d(m,n_r) = 12 \cdot w_o \cdot \left(\frac{m^2 \cdot k_d^2 \cdot n_r^2}{64 \cdot (m^2 + 4) + n_r^2} \right)^2 \qquad (1\text{-}27)$$

The total energy of oscillations of quantoide is peer to the sum of all three energies

$$w(m,n_r) = w_r(m,n_r) + w_t(m,n_r) + w_d(m,n_r) \qquad (1\text{-}28)$$

In Fig.8 the relation of a total energy of radial polytron to the frequency order is shown at two values of the amplitude order. As it is follows from the diagrams in Fig.8, the total energy of oscillations of polytron is very responsive to the amplitude order (approximately under geometrical progression). But especially paradoxical it seems the fall of the total energy with increasing of the frequency order. This implies that at radial oscillations for quantoide higher frequencies are expedient energetically. The reason of such character of change of the total energy is hidden in different distribution of radial and tangential component of energy lengthwise of quantoide, as causes the reallocation of energies with change of the frequency order of polytron.

In Table 1 the relations of radial and tangential energy of quantoide and energy of dynamic layer to its full oscillatory energy are adduced at two values of the frequency order of polytron $\boldsymbol{m} = 2$ and $\boldsymbol{m} = 32$.

Table 1

Indication	m=2, n_r=√2	m=32, n_r=√2
$w_r(m,n_r)$ / $w(m,n_r)$	50.765%	98.609%
$w_t(m,n_r)$ / $w(m,n_r)$	49.098%	0.372%
$w_d(m,n_r)$ / $w(m,n_r)$	0.137%	1.019%

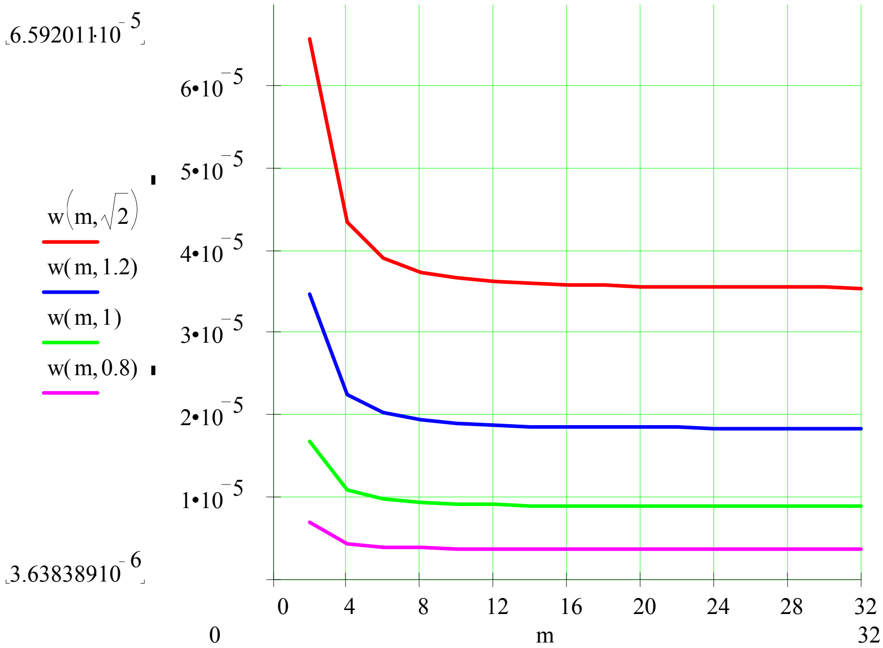

Fig. 8

Graphic chart of a total energy of radial polytron to the frequency order m at different values of the amplitude order.

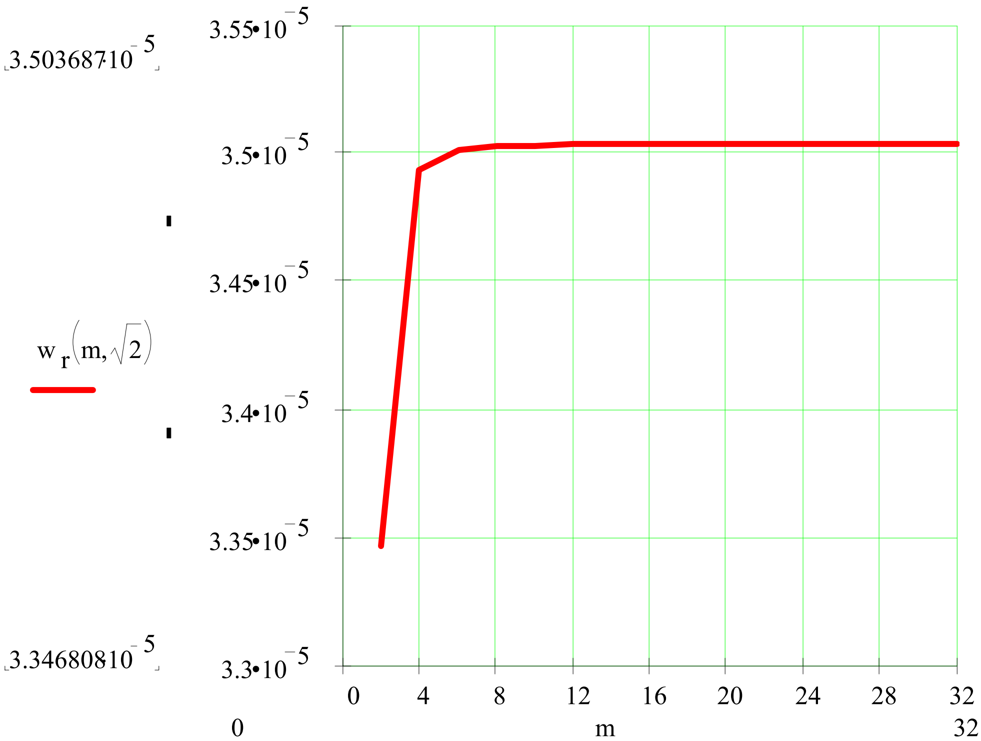

Fig. 9

Graphic chart of radial energy of polytron from the frequency order m.

In Fig.9 the curves of relation of radial components of energy from the frequency order is shown separately. As it is visible from Fig.9, at initial values of the frequency order, radial polytron has the energetic trap. The radial component of energy of polytron at frequency, which appropriate to the frequency order $\boldsymbol{m} = 2$, has the minimum, and, apparently, its level determines a short-wave boundary of radiation. At obtaining of portion of energy from the outside, polytron passes into the higher energy level on the curve $\mathbf{w_r(m,n_r)}$, but at the same time it should appear on a much lower energy level on the curve $\mathbf{w_t(m,n_r)}$. The energy, which correspond to a difference of the frequency orders on the curve $\mathbf{w_t(m,n_r)}$, should be by any way thrown out from polytron. Customary, we shall tell so, the vectorial conversion of one energy to other is represented improbable. At first, in distribution of radial and tangential energies of quantoide there is no proportionality. And, secondly, the vectors of radial and tangential amplitudes always are orthogonally related, and their maxima are carried on the angle $\boldsymbol{\pi/\mathbf{m}}$. To admix radial and tangential energy of polytron, it is as good as to admix water and oil. There should be some intermediate gear of interaction between these components. Such, for example, as pressure. Then it is possible to explain process of let out from polytron of excesses of tangential energy more or less probable. The excess of radial energy (water) presses on the reserve of tangential energy (oil) and instigates its let out from polytron.

The development of these events happen in full conformity to the formulas (1-25) and (1-26). Radial and tangential energies are proportional to the fourth degree of the amplitude order. Therefore, amplitude order is a link between them and provides balance of radial and tangential energies in polytron. The radial energy, except for the lower frequencies, almost does not depend on the frequency order, whereas the tangential energy is inversely proportional of the second degree of the frequency order. Therefore, let out of energy should happen to the help of tangential oscillations, i.e. on tangent to quantoide and in points with maximum tangential amplitude, i.e. from nodes of polytron. This process will be considered in details after comparison of energy parameters of radial and axial polytrons.

The research of the equation of radial components of energy has shown, that at the small amplitude orders, with decreasing of the amplitude order the depth of the enerrgetic trap decreases. At some value of the amplitude order, the energetic trap disappears, and the curve of radial energy by a jump is degenerated into a practically straight horizontal line.

Assigning of the equation (1-17) in a more roughly approximated kind, than the formula (1-25), gives the same outcome at all values of the amplitude order.

$$w_r(m,n_r) \approx \frac{3 \cdot w_o}{2} \cdot 0.1875 \cdot k_d^4 \cdot n_r^4 \qquad \text{(1-25a)}$$