2. PHYSICAL and MATHEMATICAL MODELLING of AXIAL POLYTRON

2.1 Methods and outcomes of physical modelling.

The axial vibrations in rings were excited by the same ways, as well as radial. Unique difference was in direction of magnetic field of permanent magnet (or of electromagnet), in an air gap of which one the segment of the ring was placed.

The resonance frequencies of axial vibrations, at small values of relation of diameter of the wire to diameter of the ring, have the same values, as well as at radial oscillations. The small differences arise at very low frequencies (up to 10ГЦ.) and at high frequencies (more than 400ГЦ.). In the first case the basic error is introduced by the mechanical factors, such as a way of fastening of rings and by their vertical or horizontal arrangement. In the second case the error is accumulated because of increasing rigidity of rings and increasing absolute error of metering equipment at more high frequencies.

As to amplitudes of axial vibrations of rings, rather essential differences from radial oscillations here are watched.

At first, the axial amplitude, at identical power input of swing, is significant more than radial.

Secondly, the axial amplitude has the precisely expressed upper bound, which has allowed to use in equations of axial vibrations narrower range of values of the axial amplitude order n_a.

Thirdly, lower than each value of resonance frequency there is an area, within of which one, the amplitude of axial vibrations has anomalous values.

In experiment it looks like.

At minimum of power build-up of oscillation and at motion on the scale of frequencies, from lower to upper, find any resonance frequency. Then the power is augmented and amplitude is drive to maximum value, so that to the ring the superfluous power is not supply. After, the frequency of the feeding generator is slowly diminished.

At that the smooth increasing of amplitude (approximately double, as contrasted to by maximal amplitude in the point of excitation of the resonance) is watched and then, at a some value of a driving frequency, the oscillations sharply cease. The decreasing of frequency makes 5÷7%. To excite oscillations on this "the point of breaking" at the expense of increase of power swing it is impossible. Besides, at the frequency, which appropriate to anomalous amplitude of axial vibrations, in character of oscillations of quantoide there is an instability and tendency to pass to radial oscillations. Or, in other words, at anomalous amplitude of axial vibrations of quantoide the change of polarization of amplitude begins.

And, in fourth, the axial vibrations of quantoide have not tangential amplitude. If to express absolutely precisely, the motion of points of quantoide at any moment of time is tangential, since happens, practically, on the sphere of axial dynamic diameter. But, at derivation of equations of energy of oscillations, this motion should be decomposed into axial and radial components. Therefore, the tangential amplitude, as such, in equations is absent.

Earlier it was mentioned, that the relations of radial and axial amplitudes to the frequency order ***m*** have hyperbolic character.

In connection with that, the marginal axial amplitudes are expressed more precisely, than radial, in experiment it was possible enough precisely to measure their values and to confirm hyperbolic relation of amplitude from ***m***.

In Fig.10 two curves, which reflect this relation are submitted. The small circles mark the values of normal amplitude, the small squares – of anomalous. The continuous lines show relations of amplitudes to the frequency order retrieved under the formula of axial quantoide, which one will be adduced in the following paragraph.

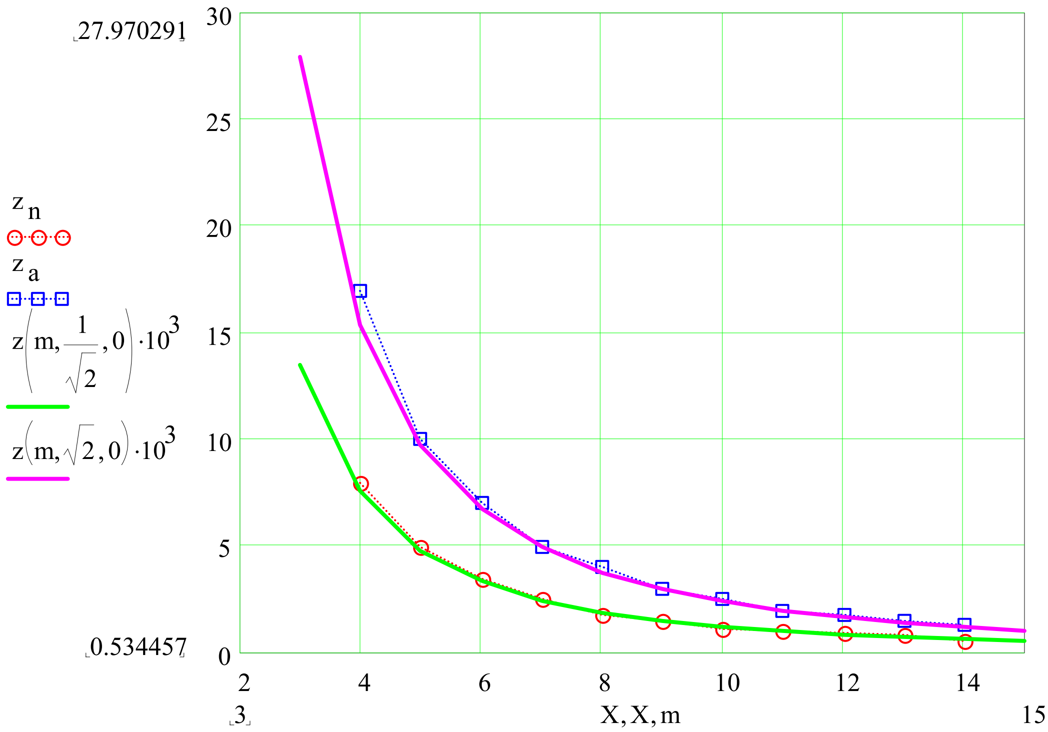

Fig. 10

Graphic chart of normal z_n and anomalous z_a axial amplitudes from the frequency order of polytron.

In Fig.11 the relations of resonance frequencies of polytron to its frequency order are shown. The mugs mark values of frequencies obtained in experiment, the dotted line shows relation of resonance frequencies to the frequency order, obtained under the formula (1-13) with the factors $\mathbf{k_m=0.4}$ and $\mathbf{k_n=0}$. The continuous line shows the same relation, obtained under the formula (1-14) with the generalized factor $\mathbf{k_o=2}$.

As it is visible from Fig.11, the curve of resonance frequencies of polytron, obtained under the formula (1-14) passes hardly above experimental curve and in parallel by last. The small reduction of the experimental curve is explained by mechanical losses of power at motion of a wire in air and, arising of effect of an added mass.

At further mathematical modelling of radial and axial polytrons the obtained value of the generalized frequency factor $\mathbf{k_o=2}$ will be used.

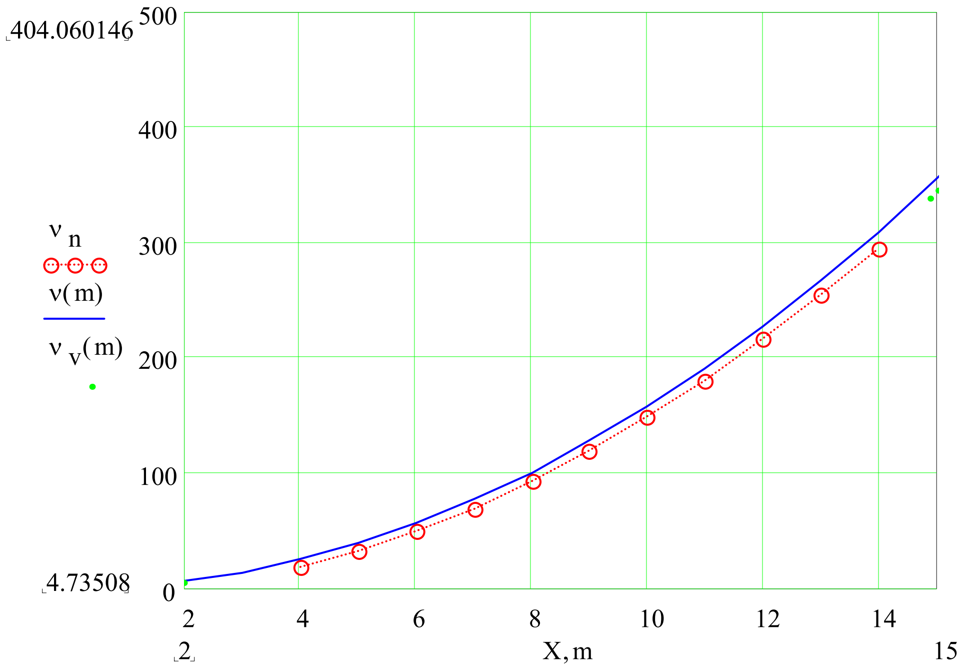

Fig. 11

Graphic chart of resonance frequencies of oscillations of quantoide from the frequency order of polytron.

2.2 Mathematical modelling of axial quantoide

As against radial polytron, where the polar radius-vector $\boldsymbol{\rho}(\mathbf{m,n_r,t,\varphi})$ is used, in axial polytron the setting of points of quantoide in time and in space is made with the help of axial amplitude $\mathbf{z(m,n_a,t,\varphi)}$. Actually, the process happens in spatial polar coordinates, but because of, the axial amplitude is connected by particular relation with polar angle $\boldsymbol{\varphi}$, the necessity of application of polar distance of spatial polar coordinates is cease. Therefore, the equation of $\mathbf{z}$-amplitude and of dynamic diameter are sufficient for the setting of points of axial quantoide.

$$z(m, n_a, t, \phi) = \frac{D_s \cdot \ln\left(\frac{1 + \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right) \cdot \cos(2 \cdot \pi \cdot \nu \cdot t)}{1 - \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right) \cdot \cos(2 \cdot \pi \cdot \nu \cdot t)}\right)}{2 \cdot m \cdot \left(1 + \frac{n_a^2 \cdot (\cos(2 \cdot \pi \cdot \nu \cdot t))^2}{6 \cdot m^2}\right)} \quad (2\text{-}1)$$

Expressions for dynamic diameter and for amplitude of boundary axial quantoide, i.e. at ***t*** = 0, receive a view

$$D_a = D_s \cdot \left[\frac{6 \cdot m^2}{6 \cdot m^2 + n_a^2}\right] \quad (2\text{-}2)$$

$$z(m, n_a, \phi) = \frac{D_a}{2 \cdot m} \cdot \ln\left(\frac{1 + \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}{1 - \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}\right) \quad (2\text{-}3)$$

The first derivative of axial amplitude on polar angle φ is described by the formula

$$z'(m, n_a, \phi) = \frac{D_a}{2} \cdot \left(\frac{n_a}{m}\right) \cdot \left(\frac{-\sin\left(\frac{m \cdot \phi}{2}\right)}{1 - \left(\frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)\right)^2}\right) \quad (2\text{-}4)$$

The projection of axial quantoide on the plane of polar coordinates is described by the end of radius-vector $\boldsymbol{\rho_a(m,n_a,\varphi)}$

$$\rho_a(m, n_a, \phi) = \frac{D_a}{2} \cdot \sqrt{1 - \frac{1}{m^2} \cdot \left[\ln\left(\frac{1 + \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}{1 - \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}\right)\right]^2} \quad (2\text{-}5)$$

The first derivative of radius-vector $\boldsymbol{\rho_a(m,n_a,\varphi)}$ on angle φ represents very cumbersome expression, therefore its denotation here is resulted only

$$\rho'_a(m, n_a, \phi) = \frac{\partial \rho_a(m, n_a, \phi)}{\partial \phi} \quad (2\text{-}6)$$

Length of boundary axial quantoide is calculated by integration from 0 up to 2π under the formula

$$l_a(m,n_a)=\int_0^{\phi}\sqrt{\left[[\rho_a(m,n_a,\phi)]^2+[\rho'_a(m,n_a,\phi)]^2+[z'(m,n_a,\phi)]^2\right]}\partial\phi \qquad (2\text{-}7)$$

The condition of persistence of length of quantoide at calculation under the formula (2-7) is maintained approximately with the same accuracy, as for radial quantoide.

Radial component of oscillations of axial quantoide is calculated as a difference between half of dynamic diameter and radius-vector $\boldsymbol{\rho_a(m,n_a,\varphi)}$

$$r(m,n_a,\phi)=\frac{D_a}{2}\cdot\left\{1-\sqrt{1-\frac{1}{m^2}\cdot\left[\ln\left(\frac{1+\frac{n_a}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)}{1-\frac{n_a}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)}\right)\right]^2}\right\} \qquad (2\text{-}8)$$

The frequency of oscillations of radial components is twice higher, than frequency of oscillations of quantoide, as well as frequency of oscillations of quantoide in dynamic layer.

The axial dynamic layer represents a hollow sphere with wall thickness

$$d_a=\frac{D_s}{2}\cdot\left(\frac{n_a^2}{6\cdot m^2+n_a^2}\right) \qquad (2\text{-}9)$$

In axial quantron, as the dynamic area, the area between the line of a circle of dynamic diameter on the plane of polar coordinates and line, circumscribed by the end of the radius-vector $\boldsymbol{\rho_a(m,n_a,\varphi)}$, is selected. It allows comparing the dynamic areas of radial and axial polytrons

$$q_a=\frac{D_a^2}{4\cdot m^2}\cdot\int_0^{\frac{\pi}{m}}\left[\ln\left(\frac{1+\frac{n_a}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)}{1-\frac{n_a}{m}\cdot\cos\left(\frac{m\cdot\phi}{2}\right)}\right)\right]^2\partial\phi \qquad (2\text{-}10)$$

The integral in equation (2-10) is enough exactly calculated with the help of the following algebraic expression

$$J_z(m,n_a)=\frac{2\cdot\pi\cdot n_a^2}{m\cdot\left(m^2-0{,}1875\cdot n_a^2\right)}\cdot\left(\frac{6\cdot m^2+n_a^2}{6\cdot m^2}\right)^2 \qquad (2\text{-}11)$$

After replacement of this integral by expression (2-11), formula of the dynamic area of axial quantron are gained by the view

$$q_a=\frac{\pi\cdot D_s^2\cdot n_a^2}{2\cdot m^3\cdot\left(m^2-0{,}1875\cdot n_a^2\right)} \qquad (2\text{-}12)$$

Accordingly, the whole dynamic area of axial polytron is equal

$$Q_a = \frac{\pi \cdot D_s^2 \cdot n_a^2}{2 \cdot m^2 \cdot \left(m^2 - 0{,}1875 \cdot n_a^2\right)} \qquad (2\text{-}13)$$

The comparison of the formula (2-13) with the formula for the dynamic area of radial polytron (1-12) gives approximated connection

$$Q_a \approx \frac{2 \cdot Q_r}{m^2} \qquad (2\text{-}14)$$

The qualitative party of the ratio (2-14) is, that at axial vibrations there is a releaser for transfers them into radial oscillations. And, with reduction of the frequency order the intensity of this gear increases, as is watched in experiment. Retroactive this gear has not, as at radial oscillations of quantoide axial component is absent and without effect from the outside to appear cannot.

2.3 Calculation of energy of oscillations axial quantoide

By taking advantage methods and equations, which applied in paragraph 1.4 we shall record equations for three energies of axial oscillations of quantoide.

Axial component of energy of axial polytron

$$u_z(m, n_a) = \frac{3 \cdot w_o}{\pi} \cdot m^3 \cdot k_d^4 \cdot \left(\frac{6 \cdot m^2}{6 \cdot m^2 + n_a^2}\right)^2 \cdot \int_0^{\frac{\pi}{m}} \left[\ln\left(\frac{1 + \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}{1 - \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}\right)\right]^2 \partial\phi \qquad (2\text{-}15)$$

By substituting in the formula (2-15) integral with its approximated algebraic expression (2-11) we shall receive more convenient formula for qualitative analysis

$$u_z(m, n_a) \approx 6 \cdot w_o \cdot \frac{m^2 \cdot k_d^4 \cdot n_a^2}{m^2 - 0{,}1875 \cdot n_a^2} \qquad (2\text{-}16)$$

Radial component of energy of axial polytron

$$u_r(m, n_a) = \frac{6 \cdot w_o}{\pi} \cdot m^5 \cdot k_d^4 \cdot \left(\frac{6 \cdot m^2}{6 \cdot m^2 + n_a^2}\right)^2 \cdot J_r(m, n_a) \qquad (2\text{-}17)$$

where

$$J_r(m, n_a) = \int_0^{\frac{\pi}{m}} \left\{1 - \sqrt{1 - \frac{1}{m^2} \cdot \left[\ln\left(\frac{1 + \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}{1 - \frac{n_a}{m} \cdot \cos\left(\frac{m \cdot \phi}{2}\right)}\right)\right]^2}\right\}^2 \partial\phi \qquad (2\text{-}18)$$

The integral (2-18) also has rather exact algebraic replacement

$$J_r(m,n_a) \approx \frac{1{,}5 \cdot \pi \cdot n_a^4}{m^7 \cdot \left(m^2 - 0{,}96 \cdot n_a^2\right)} \cdot \left(\frac{6 \cdot m^2 + n_a^2}{6 \cdot m^2}\right)^2 \quad (2\text{-}19)$$

Substituting in the formula (2-17) by replacement of integral from (2-19) we shall receive

$$u_r(m,n_a) \approx 9 \cdot w_o \cdot \frac{k_d^4 \cdot n_a^4}{m^2 \cdot \left(m^2 - 0{,}96 \cdot n_a^2\right)} \quad (2\text{-}20)$$

Energy of oscillations of dynamic layer

$$u_d(m,n_a) = 12 \cdot w_o \cdot \left(\frac{m^2 \cdot k_d^2 \cdot n_a^2}{6 \cdot m^2 + n_a^2}\right)^2 \quad (2\text{-}21)$$

The total energy of resonant oscillations of axial polytron is equal

$$u(m,n_a) = u_z(m,n_a) + u_r(m,n_a) + u_d(m,n_a) \quad (2\text{-}22)$$

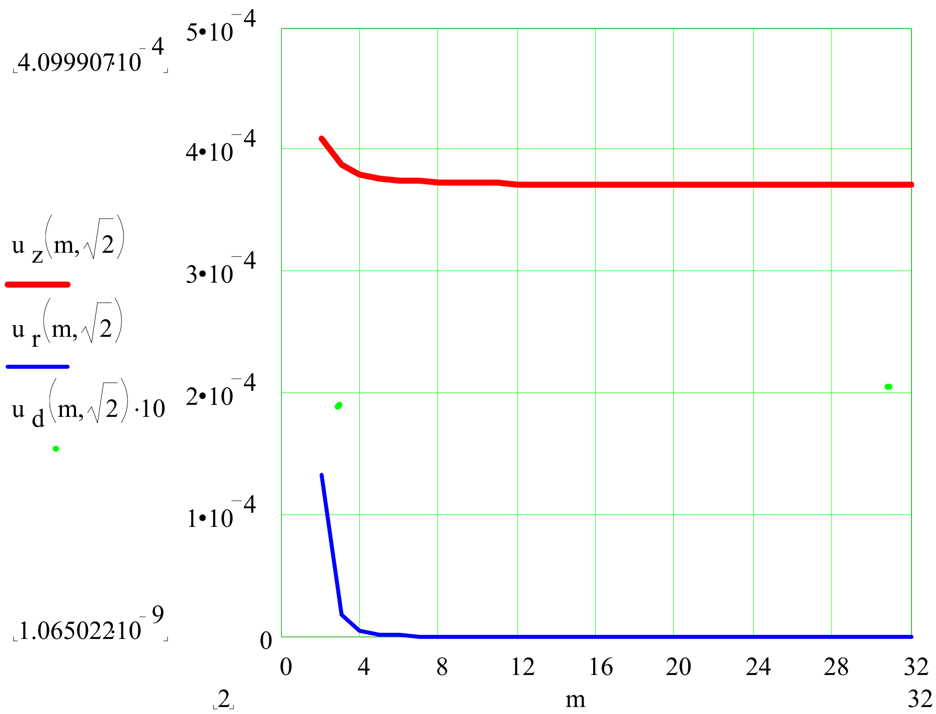

Fig. 12

Graphic chart of three components of energy of resonant oscillations of quantoide from the frequency order of axial polytron:
Solid heavy line - axial component of energy. Solid light line - radial component of energy. Dotted line - energy of dynamic layer.

In Fig.12 all three components of energy of axial polytron are submitted. And, because of small values of energy of dynamic layer, its values are multiplied by 10.

In table 2 the relation of each of three components to the total energy of polytron is adduced at values of the frequency order ***m*** = 2 and ***m*** = 32.

Table 2

Indication	m=2, $n_r=\sqrt{2}$	m=32, $n_r=\sqrt{2}$
$u_z(m,n_a)$ / $u(m,n_a)$	73.01%	94.742%
$u_r(m,n_a)$ / $u(m,n_a)$	23.558%	0.0003 %
$u_d(m,n_a)$ / $u(m,n_a)$	3.132%	5.258%

The functions, which reflect relations of different components of energy of the radial polytron (Fig.7) and of the axial polytron (Fig.12), are concerned to the same object. In this case, model of polytron is the ring of diameter **Ds = 170mm**, made from a brass wire of diameter **d = 0.27mm**. Values of energy on coordinate axises in Figs.7 and 12 are given in joules. The particular ratio between energies of radial and axial polytrons does not exist. As it is visible from the reduced charts, energy of radial and axial polytrons, at identical value of the amplitude orders differ approximately on the order. At decreasing of amplitudes, this difference increases in geometrical progression. Therefore, the role of axial polytron should differ from the role of radial polytron. Axial polytron can be the very effective storage of energy.

In Fig.13 the total energies of axial polytron are shown at different amplitude orders. Energy of axial polytron at the frequency orders ***m* = 4** and higher almost completely consists of z-component. The dynamic layer of axial polytron is volumetric and consequently has considerably large power consumption, than dynamic layer of radial polytron.

Both radial and axial polytrons energetically is more expedient to be on high resonance frequencies. Therefore, by equating the radial component of energy of radial polytron under the formula (1-25a) to the axial component of energy of axial polytron under the formula (2-16), it is possible to find equilibrium ratio between the amplitude orders of polytrons at high value of the frequency order.

This ratio has the kind

$$n_a^2 \approx \frac{0{,}1875 \cdot m^2 \cdot n_r^4}{4 \cdot m^2 + 0{,}1875^2 \cdot n_r^4} \qquad (2\text{-}23)$$

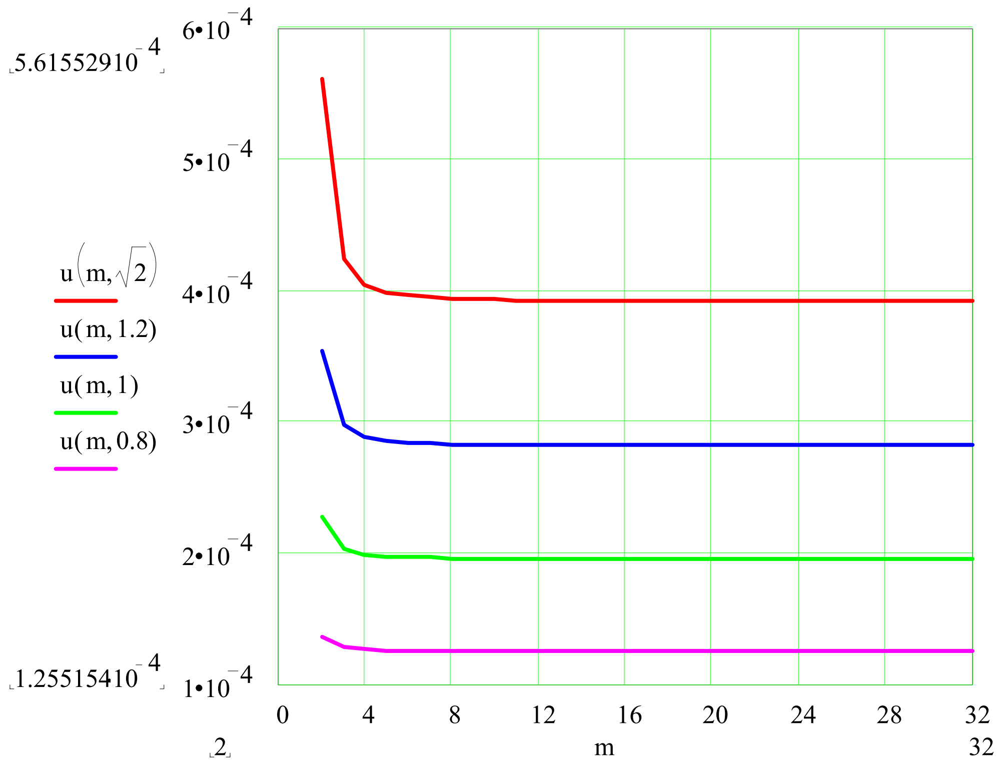

Fig. 13

Graphic chart of a total energy of axial polytron from the frequency order m at different values of the amplitude order.

At such ratio of amplitude orders, energies of polytrons, at high resonance frequencies, are compared and in experience are watched left-spiral and right-spiral volumetric polytrons. Amplitude of oscillations in such polytrons changes polarization at transition from one point of quantoide to other. The angle of polarization in each point has sensitive reaction to frequency drift of energy swing, but does not react almost to its level.

At identical width of dynamic layers, ratio of the amplitude orders of axial and radial polytrons a little diverse

$$n_a^2 = \frac{0{,}1875 \cdot m^2 \cdot n_r^2}{2 \cdot \left(m^2 + 4\right)} \qquad (2\text{-}24)$$

2.4 Function of centripetal acceleration.

Pursuant to energy postulate, ergoline has the translational component, equal to speed of light. The role of this component consists in cyclical carrying over lengthways of quantoide of energy of lateral oscillations, i.e. of radial components of energy. Or, in other words, translational component of ergoline controls by phase of radial oscillations in each point of quantoide.

The value of translational component of ergoline is constant and represents the reserve of potential energy of polytron.

To any motion on a curve trajectory is accompany a centripetal acceleration, which one is calculated, as product of a square of speed on curvature of trajectory of motion in the given point.

At changeover from mechanical oscillations of wire rings to oscillations of real polytron, in aforecited equations of energy, the factor $\mathbf{k_d}$ has sense of rigidity or elasticity of quantoide. Thus, the factor $\mathbf{k_d}$ is a function of curvature of quantoide and depends on angle $\boldsymbol{\varphi}$, and time $\boldsymbol{t}$.

The formulas for calculation of curvature $\mathbf{K(m,n_r,\varphi)}$ of boundary quantoide in its any point and centripetal acceleration $\mathbf{g_k(m,n_r,\varphi)}$ of ergoline in the given point are adduced below

$$K(m,n_r,\phi)=\frac{\rho(m,n_r,\phi)^2+2\cdot\rho'(m,n_r,\phi)^2-\rho(m,n_r,\phi)\cdot\rho''(m,n_r,\phi)}{\left[\rho(m,n_r,\phi)^2+\rho'(m,n_r,\phi)^2\right]^{\frac{3}{2}}} \quad (2\text{-}25)$$

$$g_k(m,n_r,\phi)=c^2\cdot K(m,n_r,\phi) \quad (2\text{-}26)$$

where $\boldsymbol{\rho''(m,n_r,\varphi)}$ – second derivative of the radius-vector $\boldsymbol{\rho(m,n_r,\varphi)}$ on the angle $\boldsymbol{\varphi}$.

The centripetal acceleration of ergoline in instants, when quantoide takes position of the circle of static diameter, is calculated under the simple formula

$$g_c=\frac{2\cdot c^2}{D_s} \quad (2\text{-}27)$$

In all equations for energies of polytrons (formulas 1-25, 1-26, 1-27, 2-16, 2-20, 2-21) there is the product $\mathbf{w_o\cdot(k_d)^4}$. By expressing the factor $\mathbf{k_d}$ through its basic values of $\boldsymbol{d}$ and $\boldsymbol{D_s}$ we shall receive at $\mathbf{v} = \boldsymbol{c}$

$$w_o\cdot k_d^4=\frac{\pi\cdot M_s}{256}\cdot\frac{2\cdot c^2}{D_s}\cdot\frac{d^4}{2\cdot D_s^3}\cdot k_o^2=\frac{\pi^2\cdot d^4}{3072}\cdot\beta\cdot g_c\cdot k_o^2 \quad (2\text{-}28)$$

Thus, the centripetal acceleration is included in equations for all energies of polytrons and expresses elasticity of quantoide. As against the radial polytron, the quantoide of axial polytron has variable curvature, which is directed perpendicularly to the direction of constant component of the centripetal acceleration.The value of centripetal acceleration can be serve as characteristic of gravitational properties of polytrons.